CHAPTER 5

5.3 Conditional Probability and Independence

Outcome: I will calculate and interpret conditional probabilities, use the general multiplication rule to calculate probabilities, use tree diagrams to model a chance process and calculate probabilities involving two or more events.

CONDITIONAL PROBABILITY

- The probability that one event happens given that another event is already known to have happened.
- Suppose we know that event A has happened. Then the probability that event B happens given that event A has happened is denoted by **P(B|A)**
  - **"Probability that B will occur given A occurred."**

$P(B|A)$ "given"

LET'S GO BACK TO OUR PIERCED EARS EXAMPLE

	Gender		
Pierced Ears?	**Male**	**Female**	**Total**
Yes	19	84	**103**
No	71	4	**75**
Total	**90**	**88**	**178**

- Previously, we discovered:

P(A) = P(Male) = $\frac{90}{178}$

P(B) = P(Pierced Ears) = $\frac{103}{178}$

P(A∩B) = P(Male AND Pierced Ears) = $\frac{19}{178}$

P(A∪B) = P(Male OR Pierced Ears) = $\frac{174}{178}$

	Gender		
Pierced Ears?	**Male**	**Female**	**Total**
Yes	19	84	**103**
No	71	4	**75**
Total	**90**	**88**	**178**

- From the two way table, we see that

$$P(male \mid pierced\ ears) = \frac{\#\ of\ students\ who\ are\ male\ and\ have\ pierced\ ears}{\#\ of\ students\ with\ pierced\ ears} = \frac{19}{103}$$

What if we focus on probabilities instead of numbers of students:

$$\frac{P(\overset{A}{male}\ and\ \overset{B}{pierced\ ears})}{P(\underset{B}{pierced\ ears})} = \frac{\frac{19}{178}}{\frac{103}{178}} = \frac{19}{103} = P(male \mid pierced\ ears)$$

$$\frac{19}{\cancel{178}} \cdot \frac{\cancel{178}}{103} = \frac{19}{103}$$

CALCULATING CONDITIONAL PROBABILITIES

- To find the conditional probability P(A | B), use the formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$\rightarrow$ A AND B

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

** This is on the formula sheet for the AP Exam

THE GENERAL MULTIPLICATION RULE

$$P(B|A) = \frac{P(A \cap B)}{\cancel{P(A)}} \times \cancel{P(A)}$$

$\cdot P(A)$

- The probability that events A an B both occur can be found using the **general multiplication rule**:

$$P(A \text{ and } B) = P(A \cap B) = \cancel{P(A)} * P(B\,|A)$$

$\cancel{P(A)}$

where P(B | A) is the conditional probability that event B occurs given that even A has already occurred.

*This is not on the AP Exam formula sheet, however it is essentially the conditional probability formula rearranged.

$$2 \times 6 = \frac{x}{\cancel{2}} \times \cancel{2}$$

$$12 = x$$

$$P(A) \cdot P(B|A) = \frac{P(A \cap B)}{\cancel{P(A)}} \times \cancel{P(A)}$$

$$P(A) \cdot P(B|A) = P(A \cap B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$ *

$$P(B|A) = .60$$

$$P(A) = .10$$

$$.60 = \frac{P(A \cap B)}{.10}$$

EXAMPLE

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- The Pew Internet and American Life Project find that 93% of teenagers (ages 12-17) use the internet, and that 55% of online teens have posted a profile on a social-networking site.
- Find the probability that a randomly selected teen uses the Internet, and has posted a profile.

$$\underset{.55}{P(\text{Profile} \mid \text{internet})} = \frac{\boxed{P(\text{Profile} \cap \text{internet})}}{\underset{.93}{P(\text{internet})}}$$

$$.93 \cdot .55 = \frac{P(A \cap B)}{\cancel{.93}} \cdot .93$$

$$.5115 = P(A \cap B)$$

TREE DIAGRAMS

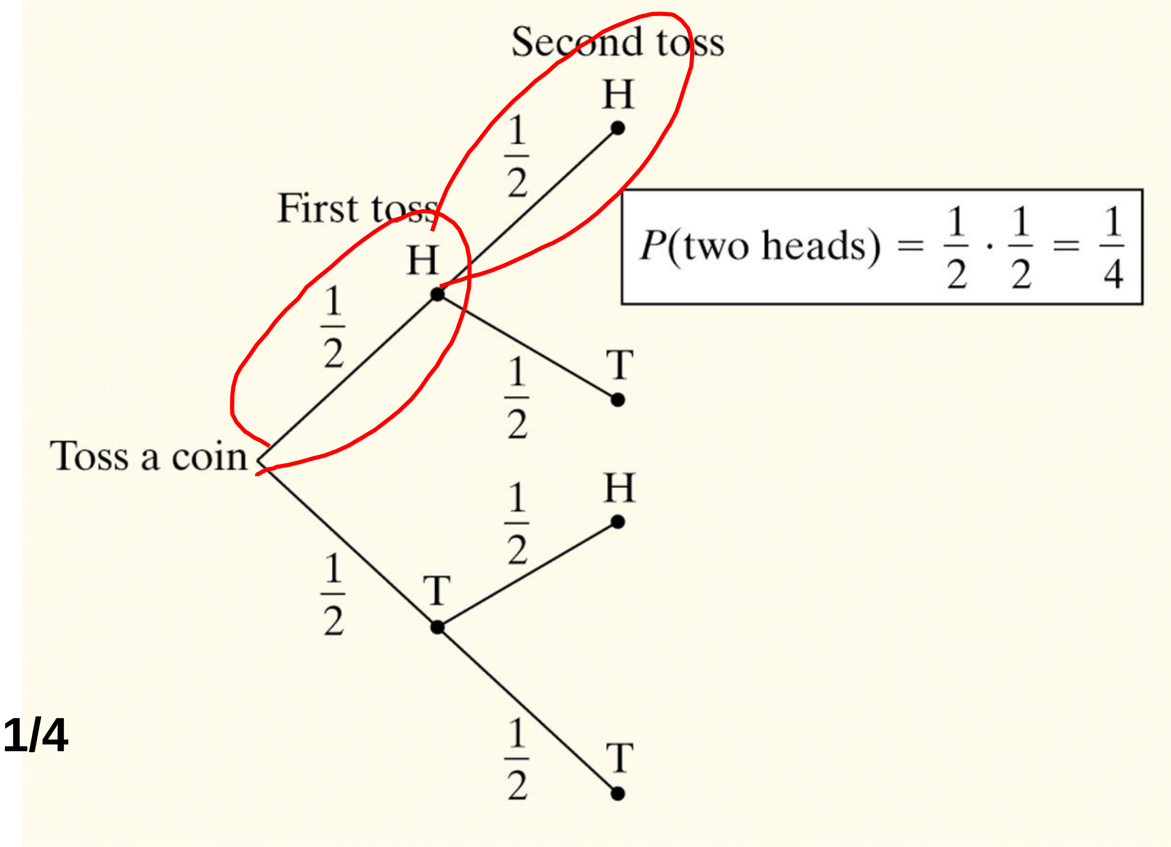
The general multiplication rule is especially useful when a chance process involves a sequence of outcomes. In such cases, we can use a **tree diagram** to display the sample space.

Consider flipping a coin twice.

What is the probability of getting two heads?

<u>Sample Space:</u>
HH HT TH TT

So, *P*(two heads) = *P*(HH) = 1/4

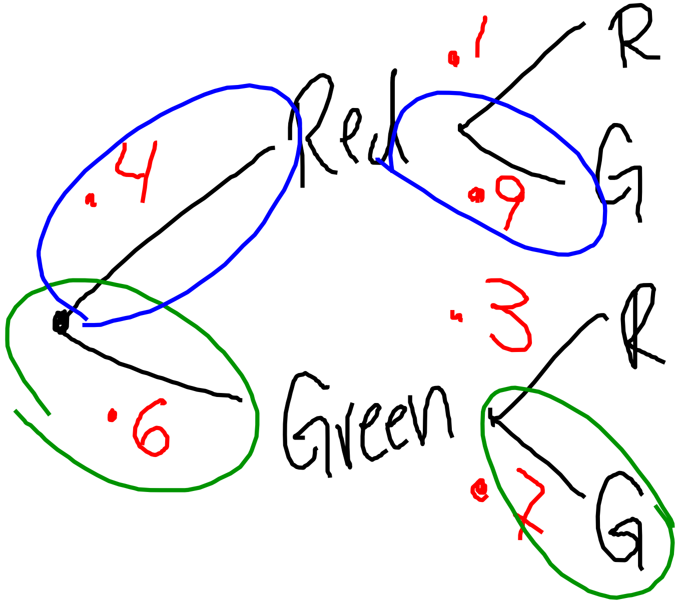


EXAMPLE

- Two consecutive traffic lights have been synchronized to make a run of green lights more likely. In particular, if a driver finds the first light to be red, the second light will be green with a probability of 0.9, and if the first light is green, the second light is to be green with a probability of 0.7. The probability of finding the first light to be green is 0.6.
- Find the probability that a driver will find the second light to be green if
  - The first light is green:

    $.6 \cdot .7 = .42$
  - The first light is red:

    $.4 \cdot .9 = .36$



HOMEWORK

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